# Tracking with Dissimilar Sensors for Linear System in Case of Arbitrary Correlated Noises

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**Abstract-** *A tracking fusion algorithm with dissimilar sensors for linear system under the condition of correlated noises is investigated based on linear unbiased minimum variance estimation theory. A new variable come from rearranging local state estimation is introduced to remove the correlation between them, and globe state update is acquired by means of sequential filtering. Not only correlated noises but also configuration difference in local sensors is involved in proposed algorithm, so information about multi-sensors fusion is increased. Through a simulation example it is indicated that the algorithm in this paper can greatly improved tracking performance.*

**Keywords:** Arbitrary correlated noises; dissimilar sensors; tracking fusion; *linear unbiased minimum variance estimation*

## 1 Introduction

In mutisensor tracking systems, the different techniques used to process the sensor's data can be grouped into two classes. The first class is called "measurement fusion" where raw sensor data is communicated to a central site for processing. The advantage of this approach is that the optimal solution can be conceivably obtained. However, due to the complexity of data association and computational requirements, this approach may not be practical. In the second class, called "state vector fusion", each sensor processes its observations locally to produce local tracks and communicates its track to a central site, where track fusion takes place. Because part of the processing takes place at the local level, the communication, track association, and computational complexities may be alleviated at the central level.

State vector fusion has been widely studied in the literature for the linear case [1]-[7]. Alouani proposed an algorithm that solves the linear distributed estimation problem using different local models where the measurement noises and processing noises are assumed to be uncorrelated [1]. Bar-Shalom refined the track fusion equations by accounting for the common process noises observed at the sensors [2]. In [3], Saha introduced a cross-correlated matrix to account for the dependence of fusing tracks. Zuo D G showed that there exist a correlation among common process noises and measurement noises [4].

This paper extends the work of [4] to an arbitrary number of dissimilar sensors case. A new variable come from rearranging local state estimation is introduced to remove the correlation between them, and globe state update is acquired by means of sequential filtering.

The problem formulation is presented in Section 2, then the proposed fusion algorithm is Section 3. Section 4 contains an example. Section 5 contains conclusion and discussion.

## 2 Problem formulation

Consider a general discrete-time linear system with arbitrary additive noise:

$$X_{k+1} = \Phi_k X_k + \Gamma_k \omega_k \tag{1}$$

$$z_k^{(i)} = H_k^{(i)} X_k + v_k^{(i)}$$

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$$i = 1, \cdots, M \tag{2}$$

where k is the time index; $X_k \in R^{n_i}$ is the state of system , $z_k^{(i)} \in R^{m_i}$ is the measurement taken by sensor $i$ ; $\Phi_k$ 、 $\Gamma_k$ and $H_k^{(i)}$ are system matrices with appropriate dimension. The process noises $\{\omega_k\}$ and the measurement noises $\{v_k^{(i)}\}$ are zero mean and colored with arbitrary covariance matrices

$$B_k^i = \mathrm{cov}(\omega_{k-1}, v_k^i) = [B_k^{(1)} \quad B_k^{(2)} \quad \cdots \quad B_k^{(i)}] \tag{3}$$

$$R_k^i = \mathrm{cov}(v_k^i) = \begin{bmatrix} R_k^{i-1} & R_k^{i-1,i} \\ (R_k^{i-1,i})^T & R_k^{(ii)} \end{bmatrix} \tag{4}$$

Such process and measurement noises will be referred to as "arbitrarily correlated noises".

Define the set of the measurements made by all sensors at time $k$ as

$$z_k^M = \{z_k^{(1)}, z_k^{(2)}, \cdots z_k^{(M)}\} \tag{5}$$

and the set of measurements available to the whole system up to time $k$ as

$$z^k = \{z_1^M, z_2^M, \cdots z_k^M\} \tag{6}$$

The coordinator problem can be stated as: Given the measurements $z^k$ , find the statistics of the global state $X_k$ .

## 3 State vector fusion algorithm with dissimilar sensors

Early work on state vector fusion was based on the assumption that the estimation errors of tracks for the same target obtained from dissimilar sensors are uncorrelated. This assumption is incorrect, however, because the process noises associated with target maneuvers is common to the filter dynamics used by the estimators. Hence, the target tracks are correlated even though the sensor measurements are uncorrelated. In this paper, a new variable come from rearranging local state estimation is introduced to remove this correlation.

Define the new variable $\rho_k^{(i)}$ as

$$\rho_k^{(i)} = \hat{X}_{k|k}^{(i)} - (I - K_k^{(i)} H_k^{(i)}) \Phi_{k-1} \hat{X}_{k-1|k-1}^{(i)}$$

$$= K_k^{(i)} z_k^{(i)} = K_k^{(i)} H_k^{(i)} X_k + K_k^{(i)} v_k^{(i)}$$

where

$$\hat{H}_k^{(i)} = K_k^{(i)} H_k^{(i)}, \quad \mu_k^{(i)} = \boldsymbol{K}_k^{(i)} v_k^{(i)}$$

then

$$\rho_k^{(i)} = \hat{H}_k^{(i)} X_k + \mu_k^{(i)}$$

$$\hat{R}_k = \mathrm{cov}(\mu_k, \mu_k) = \{\hat{R}_k^{(ij)}\}$$

$$\hat{R}_k^{(ij)} = \mathrm{cov}(\mu_k^{(i)}, \mu_k^{(j)}) = K_k^{(i)} R_k^{(ij)} (K_k^{(j)})^T$$

$$\hat{B}_k^{(i)} = \mathrm{cov}(\omega_{k-1}, \mu_k^{(i)}) = B_k^{(i)} (K_k^{(i)})^T$$

**Assumption 1:** The Best Linear Unbiased Estimate for arbitrary process and measurement noises satisfy the following equations

$$E^*[\omega_k | \rho^k] = E[\omega_k] = 0 \tag{7}$$

$$E^*[\mu_k^{(i)} | \rho^{k-1}] = E[\mu_k^{(i)}] = 0 \tag{8}$$

Then the prediction of fused track and its covariance becomes

$$\hat{X}_{k|k-1} = E^*[X_k | \rho^{k-1}]$$

$$= E^*[\Phi_{k-1} X_{k-1} + \Gamma_{k-1} \omega_{k-1} | z^{k-1}]$$

$$= \Phi_{k-1} \hat{X}_{k-1|k-1} \tag{9}$$

$$P_{k|k-1} = \mathrm{cov}\{\tilde{X}_{k|k-1}, \tilde{X}_{k|k-1}\}$$

$$= \Phi_{k-1} P_{k-1|k-1} \Phi_{k-1}^T + \Gamma_{k-1} Q_{k-1} \Gamma_{k-1}^T \tag{10}$$

The update of fused track is acquired by means of sequential filtering, where the prediction of fused track as the first sensor's prediction, then the $M$ th sensor's update recursively acquired as the globe state estimation. The first sensor local state estimation is given by

$$\hat{X}^{1}_{k|k} = E^{*}[X_k \big| \rho^{k-1}, \rho_k^{(1)}]$$

$$= \hat{X}_{k|k-1} + \hat{K}_k^{(1)}[\rho_k^{(1)} - \hat{H}_k^{(1)}\hat{X}_{k|k-1}] \tag{11}$$

where

$$\hat{K}_k^{(1)} = \hat{\Lambda}_k^{(1)}(\hat{\Xi}_k^{(1)})^{-1}$$

$$\hat{\Lambda}_k^{(1)} = \mathrm{cov}(\tilde{X}_{k|k-1}, \tilde{\rho}_k^{(1)})$$

$$\hat{\Xi}_k^{(1)} = \mathrm{cov}(\tilde{\rho}_k^{(1)})$$

$\hat{\Lambda}_k^{(1)}$ and $\hat{\Xi}_k^{(1)}$ are given by

$$\hat{\Lambda}_k^{(1)} = \mathrm{cov}(\tilde{X}_{k|k-1}, (\hat{H}_k^{(1)}\tilde{X}_{k|k-1} + \mu_k^{(1)})^T$$

$$= P_{k|k-1}(\hat{H}_k^{(1)})^T + \Gamma_{k-1}\hat{B}_k^{(1)}$$

$$\hat{\Xi}_k^{(1)} = \mathrm{cov}(\tilde{\rho}_k^{(1)}) = \hat{H}_k^{(1)}P_{k|k-1}(\hat{H}_k^{(1)})^T +$$

$$\hat{H}_k^{(1)}\Gamma_{k-1}\hat{B}_k^{(1)} + (\hat{H}_k^{(1)}\Gamma_{k-1}\hat{B}_k^{(1)})^T + \hat{R}_k^{(11)}$$

and its covariance is acquired by

$$P_{k|k}^{(1)} = \mathrm{cov}(\tilde{X}_{k|k}^{(1)}) = P_{k|k-1} - \hat{K}_k^{(1)}\hat{\Xi}_k^{(1)}(\hat{K}_k^{(1)})^T \tag{12}$$

The $i$ th sensor local state estimation is given by

$$\hat{X}^{i}_{k|k} = E^{*}[X_k \big| \rho^{k-1}, \rho_k^{i-1}, \rho_k^{(i)}]$$

$$= \hat{X}^{i-1}_{k|k} + \hat{K}_k^{(i)}(\rho_k^{(i)} - \hat{H}_k^{(i)}\hat{X}^{i-1}_{k|k}) \tag{13}$$

where

$$\hat{K}_k^{(i)} = \hat{\Lambda}_k^{(i)}(\hat{\Xi}_k^{(i)})^{-1}$$

$$\hat{\Lambda}_k^{(i)} = \mathrm{cov}(\tilde{X}^{i-1}_{k|k}, \tilde{\rho}_k^{(i)})$$

$$\hat{\Xi}_k^{(i)} = \mathrm{cov}(\tilde{\rho}_k^{(i)})$$

Using

$$\tilde{X}^{i-1}_{k|k} = X_k - \hat{X}^{i-1}_{k|k} = X_k - E^{*}(X_k \big| \rho^{k}, \rho_k^{i-1})$$

$$= (I - \hat{K}_k^{i-1}\hat{H}_k^{i-1})\tilde{X}_{k|k-1} - \hat{K}_k^{i-1}\mu_k^{i-1}$$

$$\tilde{\rho}_k^{(i)} = \rho_k^{(i)} - E^{*}[\rho_k^{(i)} \big| \rho^{k-1}, \rho_k^{i-1}]$$

$$= \hat{H}_k^{(i)}\tilde{X}^{i-1}_{k|k} + \mu_k^{(i)}$$

Then

$$\hat{\Lambda}_k^{(i)} = E^{*}[\tilde{X}^{i-1}_{k|k}\rho_k^{(i)} - \hat{H}_k^{(i)}\hat{X}^{i-1}_{k|k})^T]$$

$$= P^{i-1}_{k|k}(\hat{H}_k^{(i)})^T + E[\tilde{X}^{i-1}_{k|k}(\mu_k^{(i)})^T]$$

Define

$$\hat{\Omega}_k^{(i)} = E^{*}[\tilde{X}^{i-1}_{k|k}(\mu_k^{(i)})^T]$$

$$= (I - \hat{K}_k^{i-1}\hat{H}_k^{i-1})\Gamma_{k-1}\hat{B}_k^{(i)} - \hat{K}_k^{i-1}\hat{R}_k^{i-1,i}$$

Then

$$\hat{\Lambda}_k^{(i)} = P^{i-1}_{k|k}(\hat{H}_k^{(i)})^T + \hat{\Omega}_k^{(i)}$$

$$\hat{\Xi}_k^{(i)} = E[(\hat{H}_k^{(i)}\tilde{X}^{i-1}_{k|k} + \mu_k^{(i)})(\hat{H}_k^{(i)}\tilde{X}^{i-1}_{k|k} + \mu_k^{(i)})^T]$$

$$= \hat{H}_k^{(i)}P^{i-1}_{k|k}(\hat{H}_k^{(i)})^T + \hat{H}_k^{(i)}\hat{\Omega}_k^{(i)} + (\hat{H}_k^{(i)}\hat{\Omega}_k^{(i)})^T + \hat{R}_k^{(ii)}$$

where

$$\hat{K}_k^{i} = [(I - \hat{K}_k^{(i)}\hat{H}_k^{(i)})\hat{K}_k^{i-1} \quad \hat{K}_k^{(i)}]$$

and its covariance is given by

$$P^{i}_{k|k} = \mathrm{cov}(\tilde{X}^{i}_{k|k})$$

$$= \mathrm{cov}(\tilde{X}^{i-1}_{k|k}) + \hat{K}_k^{(i)}\mathrm{cov}(\tilde{\rho}^{i}_{k|k})(\hat{K}_k^{(i)})^T$$

$$= P^{i-1}_{k|k} - \hat{K}^{(i)}_{k} \hat{\Xi}^{(i)}_{k} (\hat{K}^{(i)}_{k})^{T} \tag{14}$$

Lastly, the global state estimation and its covariance now becomes

$$X_{k|k} = X^{M}_{k|k} \tag{15}$$

$$P_{k|k} = P^{M}_{k|k} \tag{16}$$

## 4 Simulation

This section presents an illustrative example of the filter presented above which serves primarily purpose to compare the performance of the proposed tracking fusion algorithm to the classic ones where the noises are correlated.

It is assumed that a target is tracked by Radar and IR sensor. Target state vector is defined as

$$X(k) = [x, \dot{x}, y, \dot{y}]$$

and system matrixes as

$$\Phi_k = \begin{bmatrix} 1 & T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & T \\ 0 & 0 & 0 & 1 \end{bmatrix}, \Gamma_k = \begin{bmatrix} T^2/2 & 0 \\ T & 0 \\ 0 & T^2/2 \\ 0 & T \end{bmatrix}$$

$$H^{(1)}_k = H^{(2)}_k = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}.$$

Because of IR sensor has typically higher angular resolution than Radar, but has lesser direct range resolution, so measurement accuracy of two sensors are given by

$$\sigma_{rI} = 400\,\mathrm{m}\,, \quad \sigma_{\theta I} = 0.027\,\mathrm{rad}$$

$$\sigma_{rR} = 200\,\mathrm{m}\,, \quad \sigma_{\theta R} = 0.0375\,\mathrm{rad}$$

Assume both sensors are delivering their measurements synchronously at $T = 2s$ intervals without communication delays and measurement noises are cross-correlated with each other by coefficient $\rho_{v,v} = 0.01$. The process noise is assumed common to both sensors and it is correlated with measurement noises by coefficient $\rho_{v,w} = 0.01$. The initial state is given by

$$X_0 = [10000m; 10m/s; 15000m; 15m/s]$$

and assumed uncorrelated with $\{w_k\}$ and $\{v_k\}$.

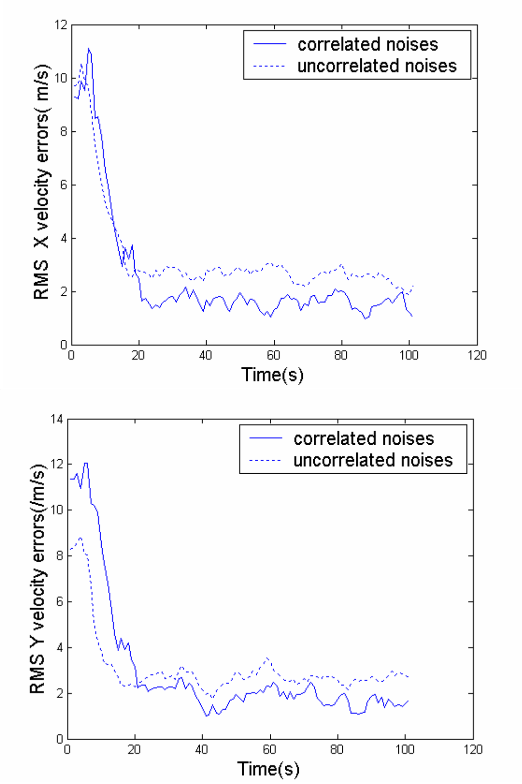


Fig.1 RMS position errors

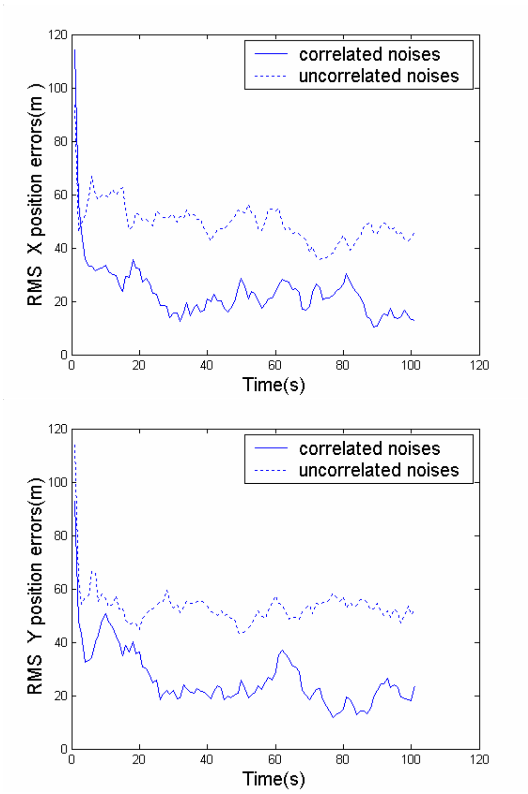


Fig.2 RMS velocity errors

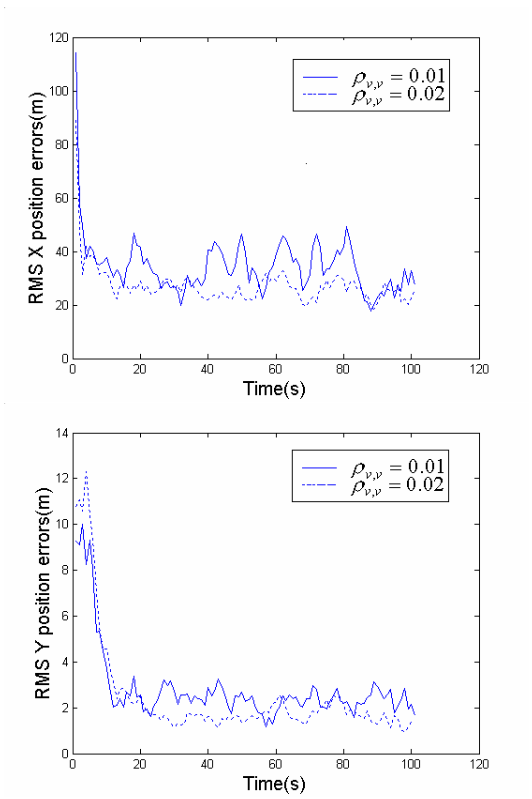

Fig.3 RMS position errors versus correlated coefficient

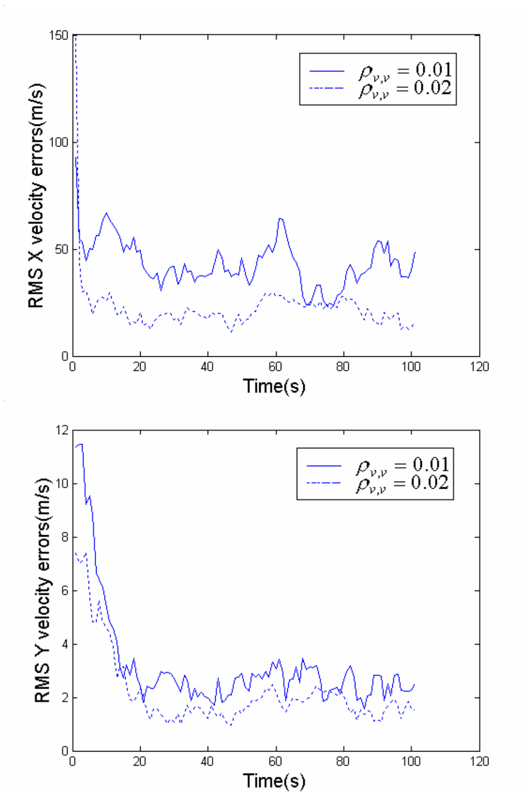

Fig.4 RMS velocity errors versus correlated coefficient

The simulation results in this section are obtained by averaging over 100 Monte Carlo runs. The simulation results are shown in Figures 1-4. Figure 1 shows the RMS position errors for both algorithms, while Figure 2 contains the RMS velocity errors. The RMS errors decrease as the correlated coefficient increase in figures 3 and 4. As can be seen from these figures the performance of the proposed tracking fusion algorithms is quite good.

## 5 Conclusions

The problem of tracking fusion with dissimilar sensors for a linear discrete-time system in arbitrary additive noises has been considered where the noises sequences have arbitrary autocorrelations. The state vector fusion algorithm is developed based on linear unbiased minimum variance estimation theory. A new variable come from rearranging local state estimation is introduced to remove the correlation between them, and globe state update is acquired by means of sequential filtering. Not only correlated noises but also configuration difference in local sensors is involved in proposed algorithm, so information about multi-sensors fusion is increased. Through a simulation example it is indicated that the results of proposed algorithm is better than that classic ones where the measurement noises and processing noises are assumed to be uncorrelated.

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